JEE MAIN April'19

PAPER WITH SOLUTION

8 April 2019 _ Evening _ Maths

1. Let $f(x) = a^x$ $(a>0)$ be written as $f(x) = f_1(x) + f_2(x)$, where $f_1(x)$ is an even function and $f_2(x)$ is an odd function. Then $f_1(x+y) + f_1(x-y)$ equals:

(1) $2f_1(x)f_1(y)$ (2) $2f_1(x+y)f_2(x-y)$
(3) $2f_1(x+y)f_1(x-y)$ (4) $2f_1(x)f_1(y)$

Sol. **1**

$$f_1(x) = \frac{a^x + a^{-x}}{2}$$

$$f_2(x) = \frac{a^x - a^{-x}}{2}$$

$$f_1(x+y) = \frac{a^{x+y} + a^{-x-y}}{2}$$

$$f_1(x-y) = \frac{a^{x-y} + a^{-x+y}}{2}$$

$$f_1(x+y)\, f_1(x-y) = \frac{a^{x+y} \quad + a^{-(x+y)} \quad + a^{(x-y)} \quad + a^{-(x-y)}}{2}$$

$= 2f_1(x)f_1(y)$

2. In an ellipse, with centre at the origin, if the difference of the lengths of major axis an minor axis is 10 and one of the foci is at $(0, 5\sqrt{3})$, then the length of its latus rectum is:

(1) 8 (2) 6 (3) 10 (4) 5

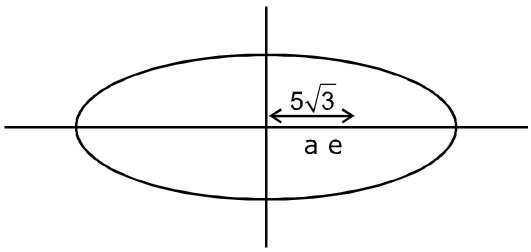
Sol. **4**

$2b - 2a = 10$
$b - a = 5$
$be = 5\sqrt{3}$
$b^2 e^2 = 25 \times 3$

$$\Rightarrow b^2 \left\{1 - \frac{a^2}{b^2}\right\}$$

$\Rightarrow b^2 - a^2 = 25 \times 3$
$\therefore b + a = 15$
$b - a = 5$
$2b = 20$
$b = 10, a = 5$
latus rectum

$$= \frac{2a^2}{b} = 5$$



3. If $z = \frac{\sqrt{3}}{2} + \frac{i}{2}\left(i = \sqrt{-1}\right)$, then $(1 + iz + z^5 + iz^8)^9$ is equal to:

(1) -1 (2) 0 (3) 1 (4) $(-1 + 2i)^9$

Sol. **1**

$$z = \frac{\sqrt{3}}{2} + \frac{i}{2}$$

$z = \cos\frac{\pi}{6} + i\sin\frac{\pi}{6}$

$z = e^{i\frac{\pi}{6}}$

$z^6 = 1$

$\left(1 + iz + \frac{1}{z} + iz^2\right)^9$

$\left(1 + e^{i\frac{2\pi}{3}} + e^{-i\frac{\pi}{6}} + e^{i(\pi/2+\pi/3)}\right)^9$

$\left(1 + \frac{1}{2} + \frac{i\sqrt{3}}{2} + \frac{\sqrt{3}}{2} - \frac{i}{2} - \frac{\sqrt{3}}{2} + \frac{i}{2}\right)^9$

$\left(\frac{1+i\sqrt{3}}{2}\right)^9$

$\left(e^{i\frac{\pi}{3}}\right)^9 = e^{i3\pi} = -1$

4. If three distinct numbers a, b, c are in G.P. and the equations $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$ have a common root, then which one of the following statements is correct?

(1) d, e, f are in A.P. (2) d, e, f are in G.P.

(3) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in A.P. (4) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in G.P

Sol. **3**

a,b, c in G.P.

say a, ar, ar^2

satisfies $ax^2 + 2bx + c = 0 \Rightarrow x = -r$

$x = -r$ is the common root, satisfies second equation $d(-r)^2 + 2e(-r) + f = 0$

$\Rightarrow d.\frac{c}{a} - \frac{2ce}{b} + f = 0$

$\Rightarrow \frac{d}{a} + \frac{f}{c} = \frac{2e}{b}$

5. Let the numbers 2, b, c be in an A.P. and $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & b & c \\ 4 & b^2 & c^2 \end{bmatrix}$. If $\det(A) \in [2, 16]$, then c lies in the interval:

(1) [4, 6] (2) $[3, 2 + 2^{3/4}]$ (3) $(2 + 2^{3/4}, 4]$ (4) [2, 3)

Sol. **1**

2b = 2 + C

|A| = (2-b) (b-c) (c-2)

$= \left(2 - \frac{2+C}{2}\right)\left(\frac{2+C}{2} - C\right)(C-2)$

$\left(\frac{2-C}{2}\right)\left(\frac{2-C}{2}\right)(C-2)$

$(C-2)^3 \in [8, 64]$

$C - 2 \in [2, 4]$

$C \in [4, 6]$

6. The number of integeral values of m for which the equation $(1 + m^2) x^2 - 2(1 + 3m)x + (1 + 8m) = 0$ has no real root is:

(1) 2 (2) 1 (3) infinitely many (4) 3

Sol. **3**

$(1+m^2)x^2 - 2(1+3m)x + (1+8m) = 0$

$D < 0$

$4\left\{-2m(2m-1)^2\right\} < 0$

$\therefore 2m(2m-1)^2 > 0$

$m > 0$

infinitely many

7. If the lengths of the sides of a triangle are in A.P. and the greatest angle is double the smallest, then a ratio of lengths of the sides of this triangle is:

(1) 5 : 6 : 7 (2) 4 : 5 : 6 (3) 5 : 9 : 13 (4) 3 : 4 : 5

Sol. **2**

$a < b < c$ are in A.P.

$\angle C = 2\angle A$ (Given)

$\Rightarrow \sin c = \sin 2A$

$\Rightarrow \sin C = 2 \sin A . \cos A$

$\Rightarrow \frac{\sin C}{\sin A} = 2\cos A$

$\Rightarrow \frac{c}{a} = 2\frac{b^2 + c^2 - a^2}{2bc}$

put $a = b - \lambda$, $c = b + \lambda$, $\lambda > 0$

$\Rightarrow \lambda = \frac{b}{5}$

$\Rightarrow a = b - \frac{b}{5} = \frac{4}{5}b, c = b + \frac{b}{5} = \frac{6b}{5}$

$\Rightarrow$ required ratio = 4 : 5 : 6

8. Given that the slope of the tangent to a curve $y = y(x)$ at any point (x,y) is $\frac{2y}{x^2}$. If the curve passes through the centre of the circle $x^2 + y^2 - 2x - 2y = 0$, then its equation is:

(1) $x^2 \log_e |y| = -2(x-1)$ (2) $x \log_e |y| = 2(x-1)$

(3) $x \log_e |y| = -2(x-1)$ (4) $x \log_e |y| = x - 1$

Sol. **2**

$m = \frac{2y}{x^2}$

$$\frac{dy}{dx} = \frac{2y}{x^2}$$

$$\frac{dy}{dx} = \frac{dx}{x^2}$$

$\frac{1}{2}\ln|y| = -\frac{1}{x} + C;$ put (1, 1)

O = – 1 + C ; C = 1

x log |y| = 2x – 2

9. A students scores the following marks in five tests : 45, 54, 41, 57 43. His score is not known for the sixth test. If the mean score is 48 in the six tests, then the standard deviation of the marks in six tests is :

(1) $\frac{100}{3}$ (2) $\frac{100}{\sqrt{3}}$ (3) $\frac{10}{\sqrt{3}}$ (4) $\frac{10}{3}$

Sol. **3**

$$\frac{45+54+41+57+43+x}{6} = 48$$

x = 288 – 240

x = 48

$$\therefore \text{s.d} = \sqrt{\sum \frac{(x-\bar{x})^2}{h}}$$

$$= \sqrt{\frac{9+36+41+81+25}{6}} = \sqrt{\frac{200}{6}} = \sqrt{\frac{100}{3}} = \frac{10}{\sqrt{3}}$$

10. Let f : R $\to$ R be a differentiable function satisfying f'(3) + f'(2) = 0. Then $\lim_{x\to 0}\left(\frac{1+f(3+x)-f(3)}{1+f(2-x)-f(2)}\right)^{\frac{1}{x}}$ is equal to:

(1) e^2 (2) 1 (3) e^{-1} (4) e

Sol. **2**

f'(3) + f'(2) = 0

$$\lim_{x\to 0}\left(\frac{1+f(3+x)-f(3)}{1+f(2-x)-f(2)}\right)^{1/x}$$

$$e^{\lim_{x\to 0}\frac{1}{x}\left\{\frac{f(3+x)-f(3)-f(2-x)+f(2)}{f(2-x)-f(2)}\right\}}$$

$$e^{\lim_{x\to 0}\left[\frac{\frac{f(3+x)-f(3)}{x}-\left(\frac{f(2-x)-f(2)}{x}\right)}{f(2-x)-f(2)}\right]} = e^{\frac{f'(3)+f'(2)}{f(2-x)-f(2)}} = e^0 = 1$$

11. If the system of linear equations x – 2y + kz = 1, 2x + y + z = 2, 3x – y – kz = 3, has a solution (x, y, z), z $\neq$ 0, then (x, y) lies on the straight line whose equation is:

(1) 4x – 3y – 4 = 0 (2) 3x – 4y – 4 = 0 (3) 4x – 3y – 1 = 0 (4) 3x – 4y – 1 = 0

Sol. **1**

Add 1st and 3rd eq.

4x – 3y – 4 = 0

12. The sum $\sum_{k=1}^{20} k\frac{1}{2^k}$ is equal to:

(1) $2 - \frac{21}{2^{20}}$ (2) $1 - \frac{11}{2^{20}}$ (3) $2 - \frac{3}{2^{17}}$ (4) $2 - \frac{11}{2^{19}}$

Sol. **4**

$\sum_{k=1}^{20} k\cdot\frac{1}{2^k}$

$S = \frac{1}{2}+\frac{2}{2^2}+\frac{3}{2^3}+.....+\frac{20}{2^{20}}$

$\frac{S}{2} = \frac{1}{2}+\frac{2}{2^2}+.....+\frac{19}{2^{20}}+\frac{20}{2^{20}}$

$\frac{S}{2} = \left(\frac{1}{2}+\frac{1}{2^2}+.....+\frac{1}{2^{20}}\right) = \frac{20}{2^{20}}$

$S = 2 - \frac{2}{2^{20}} \frac{-20}{2^{20}}$

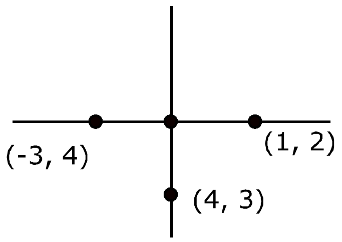
$= 2 - \frac{22}{2^{20}}$

$= 2 - \frac{11}{2^{19}}$

13. Supopose that the points (h, k), (1, 2) and (−3, 4) lie on the line L_1. If a line L_2 passing through the points (h, k) and (4, 3) is perpendicular to L_1, then $\frac{k}{h}$ equals:

(1) 0 (2) $-\frac{1}{7}$ (3) $\frac{1}{3}$ (4) 3

Sol. **3**



$y - 2 = \frac{-1}{2}(x-1)$

24 − 4 = −x + 1

x + 2y = 5

4x − 2y = 10 ...(1)

$M_t = \frac{2}{-4} = \frac{-1}{2}$

$m_n = 2$

y − 3 = 2 (x − 4)

$y - 3 = 2x - 8$
(1) and (2)
$2x - y = 5$ ...(2)
$5x = 15$
$x = 3$
$y = 1$

$\frac{K}{h} = \frac{1}{3}$

14. The number of four-digit numbers strictly greater than 4321 that can be formed using the digits 0, 1, 2, 3, 4, 5 (repetition of digits is allowed) is:
(1) 310 (2) 306 (3) 288 (4) 360

Sol. 1
0, 1, 2, 3, 4, 5

(i) [5] [ ] [ ] [ ] (6×6×6) = 216

(ii) [4] [4] [ ] [ ] (6×6) = 36

(iii) [4] [5] [ ] [ ] 6 6 = 36

(iv) [4] [3] [2] [ ] 4 = 4

(v) [4] [3] [3] [ ] 6 × 3 = 18

310

15. The height of a right circular cylinder of maximum volume inscribed in a sphere of radius 3 is:
(1) $\sqrt{3}$ (2) $\sqrt{6}$ (3) $\frac{2}{3}\sqrt{3}$ (4) $2\sqrt{3}$

Sol. 4
$r = 3\cos\theta$

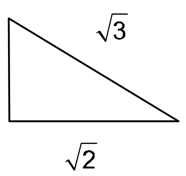
$\frac{h}{2} = 3\sin\theta$

$h = 6\sin\theta$
$V = \pi r^2 h$
$V = \pi\, 9\cos^2\theta.\ 6\sin\theta$
$V = 54\,\pi\cos^2\theta.\ \sin\theta$

$\frac{dv}{d\theta} = 54\pi\left\{\cos^3\theta - 2\cos\theta\sin^2\theta\right\}$

$\cos\theta = 0,$ $2S^2 = c^2$

$\tan^2\theta = \frac{1}{2}$



$\tan\theta = \frac{1}{\sqrt{2}}$

$h = \frac{6}{\sqrt{3}} = 2\sqrt{3}$

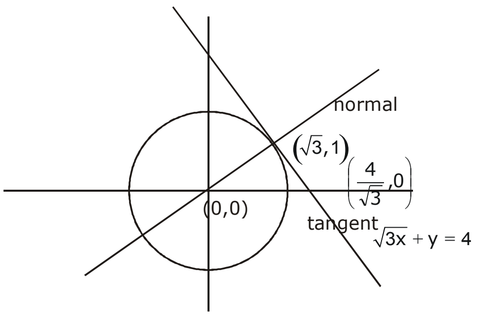
16. If f(1) = 1, f'(1) = 3, then the derivative of f(f(f(x))) + $(f(x))^2$ at x = 1 is:
(1) 33 (2) 9 (3) 15 (4) 12

Sol. **1**
f(1) = 1, f'(1) = 3
f(f(f(x)) + $f^2(x)$
f'(f(f(x)))· f'(f(x))· f'(x) + 2 f·f'
3·3·3 + 2·1·3
27 + 6 = 33

17. The tangent and the normal lines at the point $(\sqrt{3}, 1)$ to the circle $x^2 + y^2 = 4$ and the x-axis form a triangle. The area of this triangle (in square units) is:

(1) $\frac{2}{\sqrt{3}}$ (2) $\frac{1}{3}$ (3) $\frac{1}{\sqrt{3}}$ (4) $\frac{4}{\sqrt{3}}$

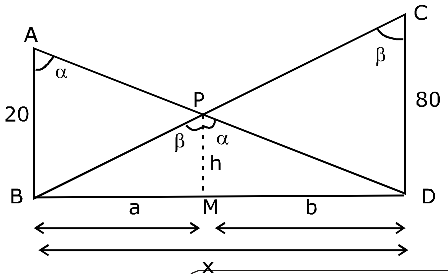
Sol. **1**



Area a = $\frac{1}{2} \times \frac{4}{\sqrt{3}} \times 1 = \frac{2}{\sqrt{3}}$

18. Two vertical poles of heights, 20 m and 80 m stand apart on a horizontal plane. The height (in meters) of the point of intersection of the lines joining the top of each pole to the foot of the other, from this horizontal plane is:
(1) 16 (2) 15 (3) 12 (4) 18

Sol. **1**



a = h tan β
b = h tan α
x = 20 tanα = 80 tanβ
Now
x = a + b
20tanα = h[tanα + tanβ]

$$h = \frac{20\tan\alpha}{\tan\alpha + \tan\beta}$$

$$h = \frac{20\tan\alpha}{\tan\alpha + \frac{\tan\alpha}{4}}$$

$$\Rightarrow \frac{80}{5} = 16$$

19. The vector equation of the plane through the line of intersection of the planes x + y + z =1 and 2x + 3y + 4z = 5 which is perpendicular to the plane x – y + z =0 is:

(1) $\vec{r}\cdot(\hat{i}-\hat{k})+2=0$ (2) $\vec{r}\times(\hat{i}-\hat{k})+2=0$ (3) $\vec{r}\cdot(\hat{i}-\hat{k})-2=0$ (4) $\vec{r}\times(\hat{i}+\hat{k})+2=0$

Sol. **1**
(x + y + z – 1) + λ (2x + 3y + 4z – 5) = 0
x(1 + 2λ) + y (1 + 3λ) + z(1 + 4λ) – 1– 5λ = 0
$\overrightarrow{n_1}\cdot\overrightarrow{n_2} = 0$
(1 + 2λ)1 – (1 + 3λ) + (1 + 4λ) = 0
1 + 3λ = 0
λ = –1/3

$$x\left(\frac{1-2}{3}\right) + y(1-1) + z\left(1-\frac{4}{3}\right) - 1 + \frac{5}{3}$$

$$\frac{x}{3} - \frac{z}{3} + \frac{2}{3} = 0$$

= x – z + 2 = 0

20. Which one of the following statements is not a tautology?

(1) $(p \wedge q) \rightarrow (\sim p) \vee q$ (2) $p \rightarrow (p \vee q)$

(3) $(p \vee q) \rightarrow (p \vee (\sim q))$ (4) $(p \wedge q) \rightarrow p$

Sol. **3**
by checking option
option 2

p	q	$p \vee q$	$p \rightarrow p \vee q$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

option 3

$p \vee q$	$p \vee \sim q$	$(p \vee q) \rightarrow p \vee \sim q$
T	T	T
T	T	T
T	F	F
F	T	T

21. If a points R(4, y, z) lies on the line segment joining the points P (2, – 3, 4) and Q(8, 0, 10), then the distance of R from the origin is :

(1) $2\sqrt{14}$ (2) 6 (3) $2\sqrt{21}$ (4) $\sqrt{53}$

Sol. **1**

$\frac{x-2}{6} = \frac{y+3}{3} = \frac{z-4}{6}$

$\frac{2}{6} = \frac{y+3}{3} = \frac{z-4}{6}$

$\frac{1}{3} = \frac{y+3}{3}$ $\frac{1}{3} = \frac{z-4}{6}$

y = – 2 z - 4 = 2

z = 6

P(4, –2, 6) distance from origin

$= \sqrt{16+4+36}$

$= \sqrt{56}$

$= 2\sqrt{14}$

22. The minimum number of times on has to toss a fair coin so that the probability of observing at least one head is at least 90% is:

(1) 4 (2) 3 (3) 5 (4) 2

Sol. **1**

Probability of observing at least one head out of n tosses

$= 1 - \left(\frac{1}{2}\right)^n \geq 0.9$

$\Rightarrow \left(\frac{1}{2}\right)^n \leq 0.1$

$\Rightarrow n \geq 4$

$\Rightarrow$ minimum number of tosses = 4

23. Let $f(x) = \int_{x+5}^{5} g(t)dt$, where g is a non-zero even function. If f(x + 5) = g(x), then $\int_0^x f(t)dt$ equals:

(1) $\int_{5+x}^{5} g(t)\,dt$ (2) $5\int_{x+5}^{5} g(t)dt$ (3) $2\int_{5}^{x+5} g(t)dt$ (4) $\int_{5}^{x+5} g(t)dt$

Sol. **1**

$$f(x) = \int_0^x g(t)\,dt$$

$$f(-x) = \int_0^{-x} g(t)\,dt$$

put t = – u

$$= -\int_0^x g(-u)\,du$$

$$= -\int_0^x g(u)\,d(u) = -f(x)$$

⇒ f (– x) = – f (x)
⇒ f(x) is an odd funtion
Also f (5+x) = g(x)
f(5 – x) = g(– x) = g(x) = f (5 + x)
⇒ f (5 – x) = f (5 + x)
Now

$$I = \int_u^x f(t)\,dt$$

$$t = u + 5$$

$$I = \int_{-5}^{x-5} f(u+5)\,du$$

$$= \int_{-5}^{x-5} g(u)\,du$$

$$= \int_{-5}^{x-5} f'(u)\,du$$

= f (x – 5) – f (– 5)
= – f (5 – x) + f (5)
f(5) – f(5 + x)

$$= \int_{5+x}^{5} f'(t)\,dt = \int_{5+x}^{5} g(t)\,dt$$

24. Let $S(\alpha) = \{(x,y) : y^2 \le x, 0 \le x \le \alpha\}$ and $A(\alpha)$ is area of the region $S(\alpha)$. If for a λ, $0 < \lambda < 4$, $A(\lambda) : A(4) = 2 : 5$, then λ equals:

(1) $2\left(\frac{4}{25}\right)^{\frac{1}{3}}$ (2) $2\left(\frac{2}{5}\right)^{\frac{1}{3}}$ (3) $4\left(\frac{4}{25}\right)^{\frac{1}{3}}$ (4) $4\left(\frac{2}{5}\right)^{\frac{1}{3}}$

Sol. **3**

$$A(4) = \int_0^4 \sqrt{x}dx$$

$$= \left(\frac{x^{3/2}}{3/2}\right)_0^4$$

$$= \frac{2 \times 4^{3/2}}{3}$$

$$= \frac{2 \times 8}{3} = \frac{16}{3} = A(4)$$

$$A(\lambda) = \frac{2}{5} \times \frac{16}{3} = \frac{32}{15} = \frac{2}{3} \times \lambda^{3/2}$$

$$\frac{16}{5} = \lambda^{3/2}$$

$$\lambda = \left(\frac{16}{5}\right)^{2/3}$$

$$= \left(\frac{256}{25}\right)^{1/3}$$

$$= 4\left(\frac{4}{25}\right)^{1/3}$$

25. If $\int \frac{dx}{x^3(1+x^6)^{2/3}} = xf(x)(1+x^6)^{\frac{1}{3}} + C$ where C is a contant of integration, then the fucntion f(x) is equal to:

(1) $-\frac{1}{6x^3}$ (2) $-\frac{1}{2x^3}$ (3) $\frac{3}{x^2}$ (4) $-\frac{1}{2x^2}$

Sol. **2**

$$\int \frac{dx}{x^3(1+x^6)^{2/3}}$$

$$\int \frac{dx}{x^3(1+x^6)^{2/3}} \qquad \frac{1}{x^6} + 1 = t^3$$

$-\frac{1}{2}\int\frac{t^2\ dt}{t^2}$ $\qquad\qquad \frac{-6^2}{x^7} = 3.t^2dt$

$-\frac{1}{2}t + e$ $\qquad\qquad \frac{dx}{x^7} = -\frac{t^2}{2}dt$

$= \frac{-1}{2}\left(\frac{1}{x^6}+1\right)^{1/3} + C$

$= \frac{1}{2x^2}\left(1+x^6\right)^{1/3} + C$

$= x\left(-\frac{1}{2x^3}\right)\left(1+x^6\right)^{1/3} + C$

$f(x) = -\frac{1}{2x^3}$

26. If the eccentricity of the standard hyperbola passing through the point (4, 6) is 2, then the equation of the tangent to the hyperbola at (4, 6) is:
(1) $3x - 2y = 0$ (2) $2x - y - 2 = 0$ (3) $x - 2y + 8 = 0$ (4) $2x - 3y + 10 = 0$

Sol. **2**

$e = 2 \qquad p(4, 6)$

$\frac{16}{a^2} - \frac{36}{b^2} = 1 \qquad ...(1)$

$e^2 = 1 + \frac{b^2}{a^2}$

$3 = \frac{b^2}{a^2} \qquad ...(2)$

$b^2 = 3a^2$

$\frac{16}{a^2} - \frac{36}{3a^2} = 1$

$\frac{4}{a^2} = 1$

$a^2 = 4$

$\therefore b^2 = 12$

$\frac{4x}{a^2} - \frac{6y}{b^2} = 1$

$\frac{4x}{4} - \frac{6y}{12} = 1$

$x - \frac{y}{2} = 1$

$2x - y = 2$

27. Let f: [−1, 3] → R be defined as $f(x) = \begin{cases} |x| + [x], & -1 \le x < 1 \\ x + |x|, & 1 \le x < 2 \\ x + [x], & 2 \le x \le 3 \end{cases}$ where [t] denotes the greatest integer less than or equal to t. Then, f is discontinuous at:

(1) only two points
(2) only three points
(3) four or more points
(4) only one points

Sol. **2**

f:[-1, 3] → R

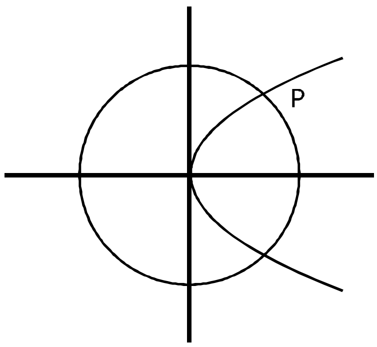
$$f(x)\begin{cases} |x| + [x] \rightarrow \begin{cases} -x-1 & x_{\in}[-1,0) \\ x & x_{\in}[0,1) \end{cases} \\ 2x & x \in [1,2) \\ x+2 & x \in [2,3) \\ x+3 & x = 3 \end{cases}$$

discontinuous at
x = 0, 1, 3

28. The tangent to the parabola $y^2 = 4x$ at the point where it intersects the circle $x^2 + y^2 = 5$ in the first quadrant, passes through the points:

(1) $\left(\frac{1}{4}, \frac{3}{4}\right)$
(2) $\left(-\frac{1}{3}, \frac{4}{3}\right)$
(3) $\left(-\frac{1}{4}, \frac{1}{2}\right)$
(4) $\left(\frac{3}{4}, \frac{7}{4}\right)$

Sol. **4**



y = 4x
$x^2 + y^2 = 5$
$x^2 + 4x - 5 = 0$
(x + 5) (x−1) = 0
x = 1
∴ y = 2
P (1, 2)
tangent at g
y = 2 (x + 1)
x − y + 1 = 0

29. Let $\vec{a} = 3\hat{i} + 2\hat{j} + x\hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$, for some real x. Then $|\vec{a} \times \vec{b}| = r$ is possible if:

(1) $r \geq 5\sqrt{\frac{3}{2}}$

(2) $0 < r \leq \sqrt{\frac{3}{2}}$

(3) $3\sqrt{\frac{3}{2}} < r < 5\sqrt{\frac{3}{2}}$

(4) $\sqrt{\frac{3}{2}} < r \leq 3\sqrt{\frac{3}{2}}$

Sol. **A**

$$\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ 3 & 2 & x \\ 1 & -1 & 1 \end{vmatrix} = (2 + x)i + j(x - 3) - 5k$$

$$|\vec{a} \times \vec{b}| = \sqrt{2x^2 - 2x + 38}$$

$$= \sqrt{2} \times \sqrt{\left(x - \frac{1}{2}\right)^2 + \frac{75}{4}}$$

$$\therefore r \geq 5\sqrt{\frac{3}{2}}$$

30. If the fourth term in the binomial expansion of $\left(\sqrt{\frac{1}{x^{1+\log_{10} x}}} + x^{\frac{1}{12}}\right)^6$ is equal to 200, and x > 1, then the value of x is:

(1) 100 (2) 10 (3) 10^3 (4) 10^4

Sol. **2**

$$T_4 = {}^6C_3 \left(x \frac{1}{1 + \log_{10} x}\right)\left(x \frac{1}{12}\right)^3 = 200$$

$$(\log_{10} x)\left(\frac{3/2}{1 + \log_{10} x} + \frac{1}{4}\right) = \frac{200 \times 6}{6 \times 5 \times 4}$$

$$\left[\frac{3}{2(1 + \log_{10} x)} + \frac{1}{4}\right](\log_{10} x) = \log_x 10$$

$$\left(\frac{3}{2(1 + t)} + \frac{1}{4}\right)t = \frac{1}{t}$$

$$\frac{(12 + 2t + 2)}{8(1 + t)} = \frac{1}{t}$$

$t(t^2 + 7t) = 4t + 4$
$t^3 + 7t^2 - 4t - 4 = 0$
$t^2 (t - 1) - 6t (t - 1) - 4(t-1)$
$t^3\ t^2 (t - 1) + 8t (t - 1) + 4 (t - 1)$
$(t - 1) (t^2 + 8t + 4) = 0$
$t = 1$
$\log_{10}x = 1$
$x = 10$

मोशन ने बनाया साधारण को असाधारण

JEE Main Result Jan'19

4 RESIDENTIAL COACHING PROGRAM (DRONA) STUDENTS ABOVE 99.9 PERCENTILE

Total Students Above 99.9 percentile -	**17**
Total Students Above 99 percentile -	**282**
Total Students Above 95 percentile -	**983**

% of Students Above 95 percentile $\frac{983}{3538}$ = **27.78%**

Scholarship on the Basis of 12th Class Result

Marks PCM or PCB	Hindi State Board	State Eng OR CBSE
70%-74%	30%	20%
75%-79%	35%	25%
80%-84%	40%	35%
85%-87%	50%	40%
88%-90%	60%	55%
91%-92%	70%	65%
93%-94%	80%	75%
95% & Above	90%	85%

New Batches for Class 11th to 12th pass
17 April 2019 & 01 May 2019

हिन्दी माध्यम के लिए पृथक बैच

Scholarship on the Basis of JEE Main Percentile

Score	JEE Mains Percentile	English Medium Scholarship	Hindi Medium Scholarship
225 Above	Above 99	Drona Free (Limited Seats)	
190 to 224	Above 97.5 To 99	100%	100%
180 to 190	Aboev 97 To 97.5	90%	90%
170 to 179	Above 96.5 To 97	80%	80%
160 to 169	Above 96 To 96.5	60%	60%
140 to 159	Above 95.5 To 96	55%	55%
74 to 139	Above 95 To 95.5	50%	50%
66 to 73	Above 93 To 95	40%	40%
50 to 65	Above 90 To 93	30%	35%
35 to 49	Above 85 To 90	25%	30%
20 to 34	Above 80 To 85	20%	25%
15 to 19	75 To 80	10%	15%

सैन्य कर्मियों के बच्चों के लिए 50% छात्रवृत्ति | प्री-मेडिकल में छात्राओं को 50% छात्रवृत्ति

Fee ₹ 1500

FOR TARGET MAY 2019 ADVANCED ASPIRANTS

Score Above 99 percentile in Jan 2019 attempt free of cost